DSP: Lec2

Given $x(n) = \{ \underset{-2}{1}, \underset{-1}{\tfrac{1}{2}}, \underset{0}{1}, \underset{1}{0}, \underset{2}{-1}, \underset{3}{\tfrac{1}{2}} \}$

a) write equation in terms of unit impulse

b) sketch $x(n)$ c) Find $x(-n+2)$

Sol

a) $x(n) = \delta(n+2) + \frac{1}{2}\delta(n+1) + \delta(n) - \delta(n-2) + \frac{1}{2}\delta(n-3)$

b, c → from the previous Lec.

→ classification of discrete-time systems:-

[1] static and dynamic discrete-time sys.

static (memoryless) ⇒ o/p of the system depends up on the present i/p only.

dynamic (memory) ⇒ o/p depends up on the past i/p also.

$\boxed{Ex}$ * $y(n) = 10\, x(n) \longrightarrow$ static $\Rightarrow$ Causal

* $y(n) = 15\, x^2(n) + \cos(n) \longrightarrow$ static $\Rightarrow$ Causal

* $y(n) = x(n-1) + 3\, x(n)^2 \longrightarrow$ dynamic $\Rightarrow$ Causal

* $y(n) = x(n+1) + x(n) + x(n-1) \longrightarrow$ dynamic $\Rightarrow$ non-Causal

$\longrightarrow$ if system need memory to store values in input $\longrightarrow$ dynamic (memory)

$\boxed{2}$ Causal and non Causal system :-

* Causal system $\Rightarrow$ o/p depends on the Present & Past i/p .

* non Causal system $\Rightarrow$ o/p depends on the future i/p also.

← الصفة الأساسية في الـ (non Causal) إن الخرج يعتمد على القيمة المستقبلية للدخل لكن بالإضافة أنه أيضاً يعتمد على قيم الماضي والحاضر .

→ non Causal sys ⇒ (un Realiable)

* $y(n) = x(2n)$

$n=0 \Rightarrow y(0) = x(0)$

$n=1 \Rightarrow y(1) = x(2)$ → non Causal

* $y(n) = x(n^2)$ → non Causal

$\boxed{3}$ shift variant system - shift invariant system

(time variant sys. - time invariant sys.)

* <u>time invariant system</u> ⇒ o/p ch/s don't change with shifting in time.

* <u>time variant system</u> ⇒ o/p ch/s change with shifting in time.

$\boxed{3}$ Lec2

→ for discrete time system :-

$$x(n) \longrightarrow \boxed{T} \longrightarrow y(n)$$

$y(n) = T\left[x(n)\right]$ ← معادلة ال system

$y(n,K) = T\left[x(n-K)\right]$ لو عملت shift

$$y(n-K) = y(n)\Big|_{n \to n-K}$$

→ if $y(n,K) = y(n-K)$ ⇒ time invariant system.

→ if $y(n,K) \neq y(n-K)$ ⇒ time variant sys.

Ex 1- $y(n) = x(n)\,\text{Cos}(wn)$

$$y(n,K) = x(n-K)\,\text{Cos}(wn)$$

$$y(n-K) = y(n)\Big|_{n \to n-K} = x(n-K)\,\text{Cos}(wn-K)$$

$y(n,K) \neq y(n-K)$ ⇒ time variant system.

2- $y(n) = x(n) + x(n-2)$

$$y(n,K) = x(n-K) + x(n-K-2)$$

$$y(n-K) = y(n)\Big|_{n \to n-K} = x(n-K) + x(n-K-2)$$

$$y(n-K) = y(n,K) \longrightarrow \text{time invariant system.}$$

3- $y(n) = x(-n)$

$$y(n,K) = x(-n-K).$$

$$y(n-K) = y(n)\Big|_{n \to n-K} = x(-(n-K)) = x(-n+K)$$

$$y(n-K) \neq y(n,K) \longrightarrow \text{time variant system.}$$

Ex: $y(n) = x(n^2)$

$$y(n,K) = x(n^2-K)$$

$$y(n-K) = y(n)\Big|_{n \to n-K} = x((n-K)^2)$$

$$y(n,K) \neq y(n-K) \longrightarrow \text{time variant system.}$$

$\boxed{4}$ Linear and non-Linear system:-

$$x_1(n) \longrightarrow \boxed{T} \longrightarrow y_1(n) = T[x_1(n)]$$

$$x_2(n) \longrightarrow \boxed{T} \longrightarrow y_2(n) = T[x_2(n)]$$

$$x_1(n) + x_2(n) \longrightarrow \boxed{T} \longrightarrow y_3(n) = T[x_1(n) + x_2(n)]$$

if: $y_3(n) = y_1(n) + y_2(n) \Rightarrow$ Linear system.

if: $y_3(n) \neq y_1(n) + y_2(n) \Rightarrow$ non-Linear system.

$\boxed{EX}$ $y(n) = x(n^2)$

$y_1(n) = x_1(n^2)$, $y_2(n) = x_2(n^2)$

$y_3(n) = x_1(n^2) + x_2(n^2)$

$\therefore y_3(n) = y_1(n) + y_2(n) \longrightarrow$ Linear sys.

2- $y(n) = x^2(n)$

$y_1(n) = x_1^2(n)$ $\quad$ $y_2(n) = x_2^2(n)$

$y_3(n) = [x_1(n) + x_2(n)]^2$

$y_3(n) \neq y_1(n) + y_2(n)$ → non-Linear sys.

3- $y(n) = \text{Log}[x(n)]$

→ non-Linear system.

4- $y(n) = \text{Cos}[x(n)]$ → non-Linear

$\boxed{5}$ stable and unstable system:-

stable ⇒ for every bounded i/p sequence,

there exist bounded o/p " B

[B I B O] → bounded i/p bounded o/p

$\boxed{7}$ Lec 2

$\boxed{Ex}$

1) $y(n) = x(n^2) \longrightarrow$ stable

2) $y(n) = x^2(n) \longrightarrow$ stable

3) $y(n) = 2^n x(n) \longrightarrow$ unstable

4) $y(n) = x(n) \cos(wn) \longrightarrow$ stable

5) $y(n) = n\, x(n) \longrightarrow$ unstable

$\boxed{Ex}$ $y(n) = x(-n+2)$

1. Dynamic
2. non-causal (depend on future) $y(0) = x(2)$

3. $y(n,k) = x(-n+2-k)$

$$y(n-k) = y(n)\Big|_{n \to n-k} = x(-(n-k)+2)$$
$$= x(-n+k+2)$$

$\longrightarrow$ time variant system.

4. $y_1(n) = x_1(-n+2)$, $y_2(n) = x_2(-n+2)$

$$y_3(n) = x_1(-n+2) + x_2(-n+2) = y_1(n) + y_2(n)$$

$\longrightarrow$ Linear

~~Ex~~

$\cancel{y(n) = x(n) + n\,x(n-1)}$

5) stable-system.

$\boxed{\text{Ex}}$ $y(n) = x(n) + n\,x(n+1)$

1) Dynamic   2) non-Causal

3) $y(n,K) = x(n-K) + n\,x(n+1-K)$

$y(n-K) = y(n)\Big|_{n \to n-K}$ $\cancel{x(n-K)}$

$= x(n-K) + (n-K)\,x(n+1-K)$

$y(n,K) \neq y(n-K)$ -

$\longrightarrow$ time-variant

4) Linear

5) un-stable.

$y(n) = \cos[x(n)]$

1) static

2) Causal.

3) time-invariant.

4) non-Linear.

5) stable.

Report

→ check the five properties.

1) $y(n) = \sum_{K=-\infty}^{n+1} x(K)$

2) $y(n) = x(2n)$

3) $y(n) = x(-n)$

$\boxed{10}$ Lec 2

→ Linear Convolution:-

$$x(n) = \sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)$$

→ To get $x(n)$

assume $K = n$

$$x(n) = \sum_{-\infty}^{\infty} x(n)$$

→ any discrete time signal can be expressed in terms of unit sample sequence (unit impulse).

→ For a discrete time system:-

$$x(n) \longrightarrow \boxed{T} \longrightarrow y(n) = T[x(n)]$$

$$y(n) = T[x(n)]$$

$$= T\left[\sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)\right]$$

→ The system is assumed to be Linear time invariant system (LTI)

$$y(n) = T\Big[\ldots\ldots + \{x(-2)\,\delta(n+2) + x(-1)\,\delta(n+1) + x(0)\,\delta(n) + x(1)\,\delta(n-1) + x(2)\,\delta(n-2) + \ldots\ldots\Big]$$

as the system is Linear :-

$$y(n) = \ldots\ldots + T\big[x(-2)\,\delta(n+2)\big] + T\big[x(-1)\,\delta(n+1)\big] + T\big[x(0)\,\delta(n)\big] + T\big[x(1)\,\delta(n-1)\big] + T\big[x(2)\,\delta(n-2)\big] + \ldots\ldots$$

~~x(0), x(1), (2)~~

$\ldots, x(-1), x(0), x(1), \ldots$ → constant values

$$y(n) = \cdots\cdots + x(-2)\, T\left[\delta(n+2)\right]$$

$$+ x(-1)\, T\left[\delta(n+1)\right] + x(0)\, T\left[\delta(n)\right]$$

$$+ x(1)\, T\left[\delta(n-1)\right] + x(2)\, T\left[\delta(n-2)\right] \cdots$$

$$\cancel{y(n) = \sum_{k=-\infty}^{\infty} x(k)\, T\left[\delta(n-k)\right]}$$

$$\boxed{y(n) = \sum_{x=-\infty}^{\infty} x(k)\, T\left[\delta(n-k)\right]}$$

assume

$$\delta(n) \longrightarrow \boxed{T} \longrightarrow h(n) \Rightarrow \text{unit-impulse response}$$

$$\delta(n-k) \longrightarrow \boxed{T} \longrightarrow h(n-k)$$

$$h(n) = T[\delta(n)]$$

$$h(n-K) = T[\delta(n-K)]$$

$$\boxed{y(n) = \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)}$$ Linear Convolution

→ The system response to any i/p sequence $x(n)$, Can be calculated by the Known of unit-impulse response $h(n)$.

$$y(n) = \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)$$

$x(n)$ → the i/p sequence.

$y(n)$ → the required response for the i/p $x(n)$

$h(n)$ → the Known unit impulse response

$$h(n) = y(n) \Big|_{x(n) = \delta(n)}$$

$\boxed{EX}$ $x(n) = \{\underset{\substack{\uparrow \\ n=0}}{1}, 1, 1, \underset{\substack{\uparrow \\ n=3}}{1}\}$, $h(n) = \{\underset{\substack{\uparrow \\ n=0}}{2}, \underset{\substack{\uparrow \\ n=1}}{2}\}$

* Compute the Convolution of the two sequences.

<u>sol</u>

$$y(n) = \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)$$

$$n_{start} = n_{x_{start}} + n_{h_{start}} = 0 + 0 = 0$$

$$n_{end} = n_{x_{end}} + n_{h_{end}} = 3 + 1 = 4$$

← هنعوض ب n من 0 ← 4

$$n=0 \Rightarrow y(0) = \sum_{K=-\infty}^{\infty} x(K)\, h(-K)$$

$$= \sum_{K=0}^{3} x(K)\, h(-K)$$

$$y(0) = x(0)\,h(0) + x(1)\,h(-1)$$

$$+\; x(2)\,h(-2) + x(3)\,h(-3)$$

$$y(0) = x(0)\,h(0) = 1 * 2 = \boxed{2}$$

$$n = 1 \Rightarrow y(1) = \sum_{K=0}^{3} x(K)\,h(1-K)$$

$$= x(0)\,h(1) + x(1)\,h(0) + x(2)\,h(-1)$$

$$x(3)\,h(-2)$$

$$= 1 * 2 + 2 * 1 = 4$$

$$n = 2 \Rightarrow y(2) = \sum_{K=0}^{3} x(K)\,h(2-K)$$

$$= x(0)\,h(2) + x(1)\,h(1) + x(0)\,h(0)$$

$$+\; x(1)\,h(-1)$$

$$= 2 * 1 + 1 * 2 = 4$$

$$n=3 \Rightarrow y(3)=4$$

$$n=4 \Rightarrow y(4)=2$$

$$y(n)=\{\underset{\uparrow}{2},\ 4,\ 4,\ 4,\ 2\}$$

$$\longrightarrow \quad x(n)=\{1,1,\underset{\underset{n=0}{\uparrow}}{0},1,1\}$$

$$h(n)=\{1,-2,-3,\underset{\underset{n=0}{\uparrow}}{4}\}$$

$\boxed{S.1}$

$$y(n)=\sum_{K=-\infty}^{\infty} x(K)\,h(n-K)$$

$$n_{start}=n_{x_{start}}+n_{h\,start}$$

$$=-2-3=-5$$

$$n_{end}=n_{x_{end}}+n_{h_{end}}$$

$$=2+0=2$$

$\underline{n=-5}$ $\quad y(-5) = \sum_{K=-2}^{2} x(k)\, h(n-k)$

$$y(-5) = \sum_{-2}^{2} x(K)\, h(-5-K)$$

$$= x(-2)\,h(-3) + x(-1)\,h(-4) +$$

$$x(0)\,\underbrace{h(-5)}_{\to 0} + x(1)\,\underbrace{h(-6)}_{=0} + x(2)\,\underbrace{h(-7)}_{=0}$$

$$= x(-2)\,h(-3) = 1 * 1 = 1$$

$\boxed{n=-4}$ $\quad y(-4) = \sum_{K=-2}^{2} x(k)\, h(-4-k)$

$$= x(-2)\,h(-2) + x(-1)\,h(-3) +$$

$$x(0)\,\underbrace{h(-4)}_{\to 0} + x(1)\,\underbrace{h(-5)}_{\to 0}$$

$$+ x(2)\,\underbrace{h(-6)}_{\to 0}$$

$$y(-4) = -2 + 1 = \boxed{-1}$$

$\underline{n = -3}$

$$y(-3) = -5$$

$\underline{n = -2}$

$$y(-2) = 2$$

$\underline{n = -1}$

$$y(-1) = 3$$

$\underline{n = 0} \Longrightarrow y(0) = -5$

$\underline{n = 1} \Longrightarrow y(1) = 1$

$\underline{n = 2} \Longrightarrow y(2) = 4$

$$y(n) = \{1, -1, -5, 2, 3, -5, 1, 4\}$$

$$y(n) = x(n) * h(n)$$

$\hookrightarrow$ Convolution

$$= \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)$$